On the Emulation of Synchronous Machine Dynamics by Converter-Interfaced Generators

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***Abstract*—This paper discusses the conditions that a device needs to satisfy to replicate the behavior of a conventional synchronous machine (SM) connected to a power network. The conditions pertain to the device's stored energy, time scale of response, oscillation damping, and behavior during short-circuits. Relevant remarks for devices that do/don't satisfy these conditions are discussed through an illustrative numerical example as well as through simulation results based on a modified version of the well-known WSCC 9-bus test system.**

***Index Terms*—Converter-interfaced generation, low-inertia systems, frequency stability, virtual synchronous machine (VSM).**

I. Introduction

A. Motivation

Unlike synchronous machines (SMs), converter-interfaced generators (CIGs) do not inherently provide inertia to the power grid, are often stochastic, and operate with small or no power reserves [1]. These properties pose serious challenges to the transition from a SM- to a CIG-dominated power system [2]. On the other hand, the behavior of CIGs is dictated by their control loops, and hence these resources are very flexible, since they can be designed using a broad range of control strategies to provide fast and effective regulation [3], [4].

B. Literature Review

The key role that SMs play in the dynamic performance of power systems is highly appreciated by system operators. This has motivated important efforts for the design of CIG control methods able to offer the auxiliary services conventionally provided by SMs, including inertial response, voltage and frequency regulation, and suppression of electromechanical oscillations. The application of these methods varies from the control of a single power electronic converter to the controlled aggregation of multiple heterogeneous converter-based resources [5]–[7]. Moreover, a part of these methods has explicitly aimed to replicate the dynamic response of SMs, which has led to the concept of virtual synchronous machine (VSM). The development of VSM is still in an early stage and various implementations have been proposed in the recent literature, for example, we cite [8]–[10].

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In a different vein, several recent studies have proposed analogies of SMs with different kinds of devices, and with a goal to study various problems, including frequency control, synchronization of power converters, transient stability, etc. For example, the authors in [11], [12] propose that a droop control is equivalent to a VSM, whereas in [13], the authors suggest an equivalence between a SM and a grid-forming converter. In [14], it is suggested that a phase-locked loop (PLL) used for converter synchronization is analogous to a SM. Yet another analogy is outlined in [15], where a non-uniform Kuramoto oscillator is described as equivalent to an overdamped SM.

Motivated by the above line of works, this paper discusses the validity of characterizing a non-synchronous device as equivalent to a SM. Such characterization, apart from a formal mathematical equivalence, should depend also upon a set of additional and critical constraints. A qualitative summary of these constraints as well as of the implications of their violation is complementary to the existing literature and can provide a didactic value for researchers working on the design of CIG control methods.

C. Contribution

The contributions of the paper are as follows:

- A qualitative description of the conditions that make a power electronic-based device behave like a traditional SM connected to a power system. These conditions pertain to the device's energy availability, time scale of action, damping, and response to short-circuits.
- A discussion on the ability to satisfy these conditions of devices proposed in the literature as equivalent SMs, including VSMs, droop controllers and PLLs.

D. Organization

The remainder of the paper is organized as follows. Section II recalls the mathematical analogy between a generic second-order oscillator and the classical SM model. The requirements that a device needs to fulfill to replicate the behavior of a traditional SM are presented in Section III. The case study is discussed in Section IV. Finally, conclusions are drawn in Section V.

II. Synchronous Machines as Oscillators

Let us recall the classical SM model [16]:

$$\begin{aligned} \dot{\delta} &= \Omega_b \, \omega \, , \\ 2H \, \dot{\omega} &= p_m - p_e(\delta) - D \, \omega \, , \end{aligned} \tag{1}$$

where δ (rad) is the rotor's angle and ω (pu) the rotor's speed variation with respect to the reference angular frequency; Ω_b (rad/s) is the synchronous frequency; H (s) is the SM inertia constant and D its damping factor; p_m and $p_e(\delta)$ are, respectively, the SM mechanical and electrical power output in pu, with $p_e(\delta) = e' v \sin(\delta - \theta)/X$, where e' is the SM internal emf; $\bar{v} = v\angle\theta$ is the voltage at the SM terminal bus; and X is defined as the sum of the SM transient reactance and the reactance that connects the SM to its terminal bus.

Let us rewrite (1) as follows:

$$c \, \ddot{y} + d \, \dot{y} - f(y) = 0 \, , \tag{2}$$

where $y \equiv \delta$, $c = 2H$, $d = D$, $f(y) = \Omega_b(p_m - p_e(\delta))$. The last equation describes a very well-known concept, i.e. the SM is a second-order oscillator, where the damping is determined by D and the "reluctance" to allow frequency variations is quantified by H. The block diagram of (2) is depicted in Fig. 1, where $s \in \mathbb{C}$ is the complex Laplace frequency.

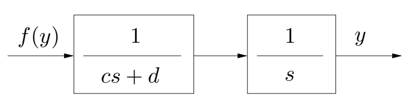


Fig. 1: Block diagram of (2).

The literature abounds of variants of (2), such as the Van der Pol [17] and the Liénard-type oscillator [18], with additional non-linear terms, e.g. $d = g(y)$, and/or forced input oscillations. We acknowledge but do not discuss these models as they can be considered to be part of the broader category of synchronization mechanisms, such as PLLs. Moreover, the very same model shown in (2) is ubiquitous in a broad class of engineering systems, which (or even more often, parts of which) can be reasonably approximated by a suitable second-order system of the same shape. Then, taking into consideration the importance of SMs in power systems, one may observe a striking equivalence of the SM with, in principle, any other system that can be described in the same form, e.g. with a given second-order automatic controller.

The main concept discussed in this paper is that, contrary to what is underlying (to lower or higher extent) in some recent works, a given device expressed in the form of (2) is, in general, not equivalent to a SM. The only obvious analogy between such a device and a SM is that they are both *special cases* of the same, broad family of oscillators. Moreover, a given device can be considered as emulating the behavior of a traditional SM connected to a power network, if and only if a set of additional constraints are met. These constraints are duly discussed in the next section.

III. Synchronous Machine Emulation

In this section we discuss the conditions that a device needs to satisfy so that it replicates the behavior of a conventional SM connected to a power grid. These conditions pertain to the availability of energy, the time scale of response, the behavior during short-circuits, and the damping of oscillations.

A. Time Scale

The time scale of the dynamic response of the emulating device must be similar to that of a SM. A typical range of the inertia time constant H in a SM is [2,10] MWs/MVA. The value of H has a physical meaning and represents the time (in seconds) for which the SM could inject its rated power to the system if disconnected from its turbine. Therefore, second-order oscillators in the form of (2) that respond in a different time frame do not resemble the behavior of a SM.

B. Stored Energy

In a SM of rated power S_n MVA, the rotating mass has in nominal conditions stored kinetic energy HS_n MWs. After a negative (positive) mismatch between the mechanical power p_m and electrical power p_e and until primary regulation is initiated, this physical storage is the crucial mechanism that maintains the system's power balance but also the SM synchronism, by decreasing (increasing) instantaneously its stored energy as the rotor decelerates (accelerates). Maintaining synchronism and power balance are inextricable in a SM, and hence, a SM-emulating device is also required to include mechanisms that account for both tasks.

Regarding the power balance, a device that emulates a SM should have sufficient stored energy that is available very fast (ideally instantaneously) after a power mismatch $\Delta p = p_m - p_e \neq 0$ occurs. "Very fast" in this context basically refers to the time delay between the occurrence of the disturbance and the initiation of the device's response. For CIGs, instantaneous (i.e. delay-free) provision of the required energy during an imbalance is not possible, and so the condition may be relaxed to a requirement for a very fast response. This however may raise concerns, as it leads to a time window right after the disturbance that remains uncovered [2]. Overall, energy storage is by no means a trivial requirement for a SM-equivalent device.

C. Oscillation Damping

Not well-damped oscillations are undesired. Thus, in an emulation of a SM where the damping is a fully controlled parameter, it is reasonable that one decides to remove oscillations during the design (e.g., in the case of (2), by choosing a large d). We recall that SMs are designed for high efficiency and thus include a relatively small damping. A typical range of D for (1) is [2,3] pu to account both for mechanical damping and effect of damper windings. In higher-order (e.g. sixth-order) machine models, the effect of damper windings is explicitly represented in the model and thus D can be chosen lower or even zero. Then, poorly damped electromechanical oscillations are partially suppressed by some form of damping control, but the resulting response is still oscillatory. In theory, good damping of SM oscillations could be achieved through prime movers, but the latter are not fast-enough due to mechanical constraints. Thus, in practice, oscillations are damped through the SM excitation system, but the effect is limited due to the weak coupling of voltage with power angle.

The above problems do not exist in power electronic-based devices, which can provide a fast response and thus also be designed for very good damping. However, it is worth noting that, an overdamped response, even if desired, does not replicate the conventional behavior of a SM connected to a power network.

D. Link of Time Scale with Energy and Damping

A qualitative way to study the link of time scale with energy and damping in model (2) is by considering its linearized version, as follows:

$$c\,\Delta\ddot{y} + d\,\Delta\dot{y} + k\Delta y = 0\,. \tag{3}$$

The variations of stored energy (ΔE) and power dissipation (ΔP_l) of the oscillator are then [19]:

$$\Delta E = \frac{1}{2}\,c\,\Delta\dot{y}^2\ ,\quad \Delta P_l = d\,\Delta\dot{y}^2\,, \tag{4}$$

while its eigenvalues are $\lambda = (-d \pm \sqrt{d^2-4ck})/2c$ and, thus, the following relationship holds:

$$\frac{\Delta E}{\Delta P_l} = \frac{c}{2d} = -\frac{1}{4\,\Re\{\lambda\}}\,. \tag{5}$$

From (5), it is clear that dynamics faster than the time scale of interest are likely to lead to high damping and also violate the requirement for available stored energy, as fast eigenvalues are in general linked to lower amounts of energy and higher damping ratios.

E. Response to Short-Circuits

The short-circuit current that a SM can tolerate before protections are activated is multiple of the nominal for some time. The high "thermal inertia" of SMs is in contrast to the limited ability to overload power converters. This implies that, even if a CIG is controlled to reproduce well the response of a SM under small disturbances, the same can not be achieved during severe voltage drops, unless the converter design is significantly overrated (e.g. by 6 to 7 times). However, such a design is not practical for economical reasons. This appears to be a rather severe limitation of VSMs in general, given that replication of the behavior of SMs is of upmost importance during large disturbances such as faults.

F. Remarks

The following remarks are relevant:

- The conditions discussed above focus mainly on the critical (for low-inertia systems) time scale of the SM inertial response, which is also the most relevant time scale for the emulation of SM dynamics. Slower actions, including primary and secondary frequency regulation, are not a concern, since they can be conveniently implemented with standard controllers without the need to make any analogy with a SM.
- In the classical model (1) the internal emf e' is constant, which makes the SM an ideal voltage source. In normal operating conditions, SMs are not ideal sources but do regulate the voltage magnitude at their terminal bus. This capability is not intrinsic of the SM *per se*, e.g., permanent-magnet SMs are unable to provide this control. On the other hand, in practice, SM-emulating devices are expected to provide voltage regulation. This is the case of CIGs controlled through a grid-forming strategy, or more precisely, voltage-forming current-following control [20]. We note, however, that the specific property of the SMs is to be *frequency-forming* – which is a consequence of (1) – not necessarily voltage-forming.

IV. Case Study

In this section we discuss through numerical simulations the behavior of devices that have been proposed in the literature as analogous and/or equivalent to SMs. Section IV-A is based on the simplified model (2), while Section IV-B is based on the well-known WSCC 9-bus test system.

A. Illustrative Example

In this section we consider different devices modeled as second-order oscillators in the form of (2). The first device is a conventional SM. The second device is a droop-based control that acts in the time scale of the SM inertial response. Since droop controls are in general not oscillatory, modeling such device with (2) implies that the oscillator is overdamped, or equivalently, that d is relatively large. We note that energy availability is not a given for droop controllers. Assuming a droop control combined with sufficient power reserve availability that can be used very fast following a disturbance is under certain conditions what has been often defined in the recent literature as VSM. The last device considered is a simplified PLL. The PLL is much faster than a SM and also does not have the required energy to provide inertial response. On the other hand, a PLL may oscillate, although the damping ratio of PLL oscillations is not necessarily similar to that of SM oscillations. Table I summarizes how droop control, VSM, and PLL compare to a conventional SM connected to a power system in view of the conditions for energy availability, time scale, damping, and short-circuit response, discussed in Section III.

TABLE I: Comparison of droop control, VSM, and PLL, with conventional SM.

Device	Energy	Time scale	Damping	Short-circuit response
Conventional SM	✓	✓	✓	✓
Droop control	✗	✓	✗	✗
VSM	✓	✓	✗	✗
PLL	✗	✗	✗	✗

Figure 2 shows how the step responses of the SM, VSM, and PLL, compare to each other. The top panel shows the frequency variations of the devices, while the bottom provides a close-up of the same plot. The values of c and d used for each device are given in Table II. These values yield the following ratios between stored energy and power dissipation in (5) for the three devices:

- SM: $\Delta E/\Delta P_l = 1$.
- VSM: $\Delta E/\Delta P_l = 0.03$.
- PLL: $\Delta E/\Delta P_l = 0.00167$.

The $\Delta E/\Delta P_l$ ratio for the SM is two and three, respectively, orders of magnitude larger than those of the VSM and PLL. This result, as well as the plots in Fig. 2, are illustrative of the significant difference between the conventional behavior of SMs in power systems and the responses of devices that do not satisfy the constraints described in Section III.

TABLE II: Parameters of second-order oscillators.

Device	SM	VSM	PLL
c	6	6	0.01
d	3	100	3

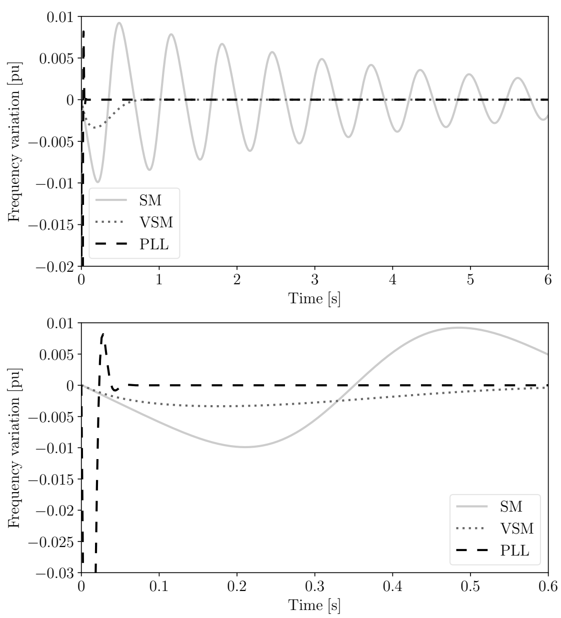
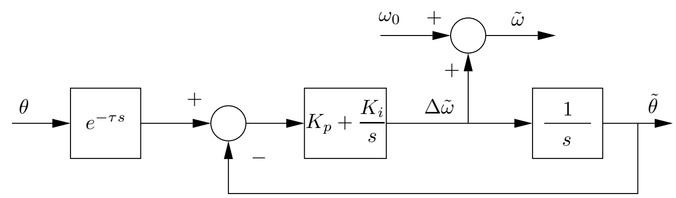


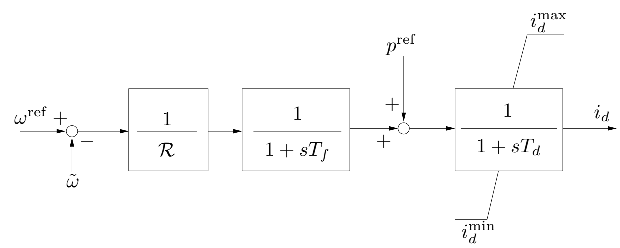
Fig. 2: Response of SM, VSM, and PLL.

B. WSCC 9-Bus System

This section is based on the WSCC 9-bus test system. The network comprises six transmission lines and three medium voltage/high voltage transformers; during transients, loads are modeled as constant admittances; two SMs are connected to buses 1 and 2, while, for the needs of this paper, the SM at bus 3 is replaced by a CIG. The modified test system is shown in Fig. 4. The CIG at bus 3 synchronizes to the power grid through a synchronous reference frame PLL and provides primary frequency response through a droop-based controller that receives the error between the reference and estimated by the PLL frequency $\omega^{\mathrm{ref}} - \tilde{\omega}$ and regulates the d-axis current component i_d in the dq-reference frame [4]. The estimated by the PLL frequency is obtained through a proportional-integral control whose input is the error between the measured and the estimated phase angles $\theta - \tilde{\theta}$. The block diagrams and parameter values of the PLL and frequency control models are presented in Fig. 3 and Table III.



(a) Synchronous reference frame PLL.



(b) Frequency control.

Fig. 3: CIG synchronization and frequency control loops.

TABLE III: CIG control parameters.

PLL	$K_p = 0.1$, $K_i = 0.05$
Frequency	$\mathcal{R} = 0.05$, $T_f = 1.2$ s, $T_d = 0.6$ s

The dynamic order of the CIG model, including the droop-based control, d and q axis current control, and PLL dynamics, is 5. The dynamic order of each SM, including frequency and voltage regulators, is 11.

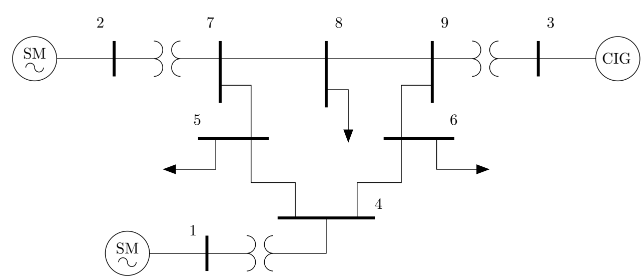


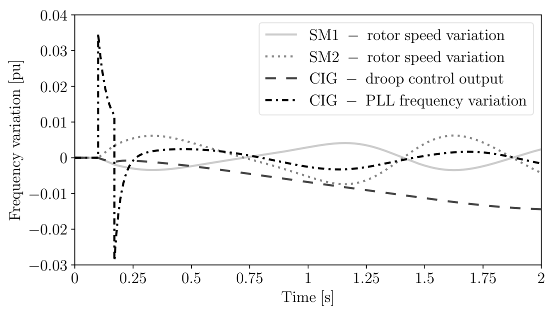
Fig. 4: Modified WSCC 9-bus test system.

Small-signal stability analysis shows that the system's electromechanical oscillation is represented by the complex pair of eigenvalues $-0.2135 \pm \jmath 8.6897$, with natural frequency 1.38 Hz and damping ratio 2.46%. Comparison of the modal participation, see [21], in this mode of the CIG droop-based control state variable, the CIG's PLL angle estimation state variable, as well as of the rotor speeds of the SMs at buses 1 (SM1) and 2 (SM2), is provided in Table IV. These results indicate that, as expected, the PLL and droop control variables are decoupled from the electromechanical mode.

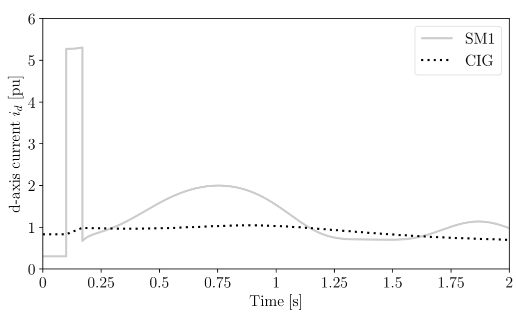
TABLE IV: Participation factors for electromechanical mode.

Variable	Participation factor
Rotor speed of SM1	0.16
Rotor speed of SM2	0.32
CIG droop control state	0.00
PLL angle estimation state	0.00

We consider a three-phase fault at bus 5 at $t = 0.1$ s. The fault is cleared after 70 ms, by tripping the line that connects buses 5 and 7. Results are summarized in Fig. 5. In particular, Fig. 5a shows the speed variation of the SMs; the (normalized) output of the CIG droop control; and the frequency variation at bus 3 as estimated by the PLL. Figure 5b illustrates the tight limit in the ability to overload the CIG and thus to support the system during the fault, by comparing the d-axis current component of the stator of the SM at bus 1 to the d-axis regulated current component of the CIG. Once again, results are representative of the large qualitative deviations between the behavior of a conventional SM connected to a power system and devices that are not designed to resemble its dynamics according to the conditions discussed in Section III.



(a) Frequency variation.



(b) d-axis current.

Fig. 5: Response following the three-phase fault, 9-bus system.

V. Conclusions

The paper shows that a given second-order oscillatory device resembles the dynamic response of a SM only if it satisfies certain conditions. These conditions are concerned with the device's availability of energy, time scale of action, damping of oscillations, and response during short-circuits. Devices that do not fulfill these conditions have been characterized in the recent literature as equivalent or analogous to SMs. Such devices should not be confused with and/or misinterpreted as replicating the traditional behavior of a SM connected to a power network.

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